INDIAN MARITIME UNIVERSITY

(A CENTRAL UNIVERSITY, GOVT. OF INDIA)

SEMESTER- II, B.TECH. (MARINE ENGINEERING) – JUNE 2014 EXAMS

MATHEMATICS - II (T 2202)

(AY 2013-14 batch onwards)

Time:- 3 Hrs | **Max Marks : 100**
Date: 20.06.2014 | **Pass Marks : 50**

PART - A (3 X10 = 30 Marks)

Compulsory Questions

1. a) Find the Fourier Coefficients a_o a_n of the function. $f(x) = \begin{cases} 0, & -5<x<0 \\ 3, & 0<x<5 \end{cases}$

b) A random variable X has the following pdf. $f(x) = \begin{cases} cx^2, & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$

Find (i) C, (i) P $(0 \le x \le \frac{1}{2})$

c) If 20% of the articles produced by a machine are defective, determine the probability that out of the 4 articles chosen at random less than 2 articles will be defective.

d) If the chance of being killed by flood during a year is $\frac{1}{3000}$, use Poisson distribution to calculate probability that out of 3000 persons living in a village, at least one will die in flood in a year.

e) If f(t) = $\begin{cases} 1 & if\ 0<t<2 \\ 2 & if\ t>2 \end{cases}$ Find the Laplace transform of f(t)

f) $If\ L\{f(t)\} = \frac{p^2 - p + 1}{(2p+1)^2(p-1)}$, apply the change of scale property to show that

$L\{f(2t)\} = \frac{p^2 - 2p + 4}{4(p+1)^2(p-2)}$

g) Find $L^{-1}\left(\frac{1}{4s^2+9}\right)$

DE h) Solve : $(x^2 + y^2 + 2x)dx + xydy = 0$

i) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^{3x}$

DE j) Find the wronskian of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$ in course of solving it by method of variation of parameters.

PART - B (5 X14 = 70 Marks)

Answer Any Five of the following

2. a) Find the Fourier Series for the function f(x) if

$$f(x) = -\pi, -\pi < x < 0$$
$$= x, 0 < x < \pi$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$

b) Find the Fourier series of $f(x) = |x|$ in $-\pi < x < \pi$ Deduce that

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ... + \frac{1}{(2n-1)^4} + ...to\,\infty = \frac{\pi^4}{96}$$

(6+8)

3. a) Three identical boxes I, II and III contain respectively 4 white and 3 red balls, 3 white and 7 red balls, 2 white and 3 red balls. A box is chosen at random and a ball is drawn out of it. If the ball is found to be white, what is the probability that box II is selected.

b) A continuous r.v. x has the p.d.f.

$$f(x) = \frac{x}{2}, \quad 0 \le x \le 1$$
$$= \frac{1}{2}, \quad 1 < x \le 2$$
$$= \frac{3-x}{2}, \quad 2 < x \le 3$$

Find (i) mean of x (ii) variance of x. (7+7)

4. a) A discrete r.v.x has the mean 6 and variance 2. Assuming the distribution is Binomial, find the probability that $5 \le X \le 7$.

b) the length of the life of a tyre manufactured by a company follows a continuous distribution given by the density function. $f(x) = \begin{cases} \frac{k}{x^3}, & 1000 \le x \le 1500 \\ 0, & elsewhere \end{cases}$

Find K and find the probability that a randomly selected tyre would function for at least 1200 hrs. (6+8)

$$\frac{1}{s^2}\left(1+e^{-2s}-e^{-s}\right)$$

5. a) Find the Laplace transform of a periodic function f(t) given by

$$f(t) = t \text{ for } 0 < t < c$$
$$= 2c - t \text{ for } c < t < 2c$$

b) Find the Laplace transform of $\int_0^t \int_0^t \int_0^t (u \sin u)\, du\, du\, du$ (6+8)

6. a) Find the inverse Laplace transform of $\dfrac{1}{(s-1)^2(s-2)^3}$ by

Convolution theorem.

b) Solve by Laplace transform the equation $y^{11}(t) + y(t) = 8 \cos t$,

where $y(0) = 1$, $y^1(0) = -1$ (6+8)

DE

7. a) Solve the following differential equation (Any three)

(i) $\dfrac{dy}{dx} + \dfrac{y \log y}{x} = \dfrac{y(\log y)^2}{x^2}$

(ii) $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = Sin2\{log(1+x)\}$

(iii) $(D^2 - 4D + 3)y = 2xe^{3x} + 3e^x \cos 2x$

(iv) $x(x-y)\, dy + y^2 dx = 0$

b) Apply the method of variation of parameters to solve the equation

$(D^2 - 2D + 1)y = e^x \log x \ \ where D \equiv \dfrac{d}{dx}$ (3x3+5)

DE

8. a) Solve the equation $L\dfrac{di}{dt} + Ri = E_0 Sin\, wt$ Where L, R, E_0 are constants

and discuss the case when t increases indefinitely.

b) Solve : $(x^7 y^2 + 3y)dx + (3x^8 y - x)dy = 0$ (7+7)

9. a) Expand $f(x) = x \sin x$ in Fourier series in the interval $0 < x < 2\pi$

(P) b) The overall percentage of failures in a certain examination is 40.

What is the probability that out of a group of 6 candidates at least 4 passed the examination. (8+6)

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